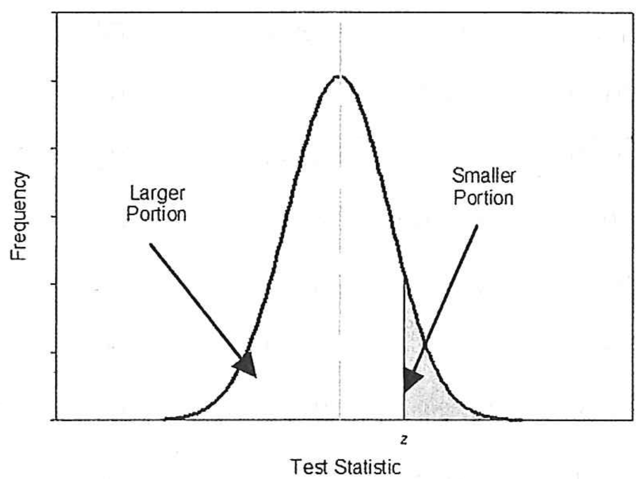
Dr. Peggy Kern's Capstone Statistics
Practice #2: The Normal Distribution & Z Scores

1. Probability values range from _0.00__ to __1.00___.

2. What is the difference between an empirical and probability distribution?
   An empirical distribution is based on observations; a probability distribution (also called a theoretical distribution) is based on logic or mathematical formulas.

3. Below is part of a normal distribution table[1]:



z	Larger Portion	Smaller Portion	y
.00	.50000	.50000	.3989
.01	.50399	.49601	.3989
.02	.50798	.49202	.3989
.03	.51197	.48803	.3988
.04	.51595	.48405	.3986
.05	.51994	.48006	.3984
.06	.52392	.47608	.3982
.07	.52790	.47210	.3980

   a. What does the first column (z) refer to?
      The z score

   b. What does the second column (larger portion) refer to?
      The area (or proportion of scores) that lie below the Z score. Note that for many online tables, this is the only part included.

   c. What does the third column (smaller portion) refer to?
      The area (or proportion of scores that lie beyond the z score. Note that this is excluded from many online tables.

4. The standard normal distribution (Z distribution) is a probability distribution with a mean of 0 and a standard deviation of 1. We can compare raw scores from different scales by converting them to Z scores (that is, standardizing the values). Recall that **z = (X-M)/s** (where **X** = the score, **M** is the mean of the sample, and **s** is the standard deviation.

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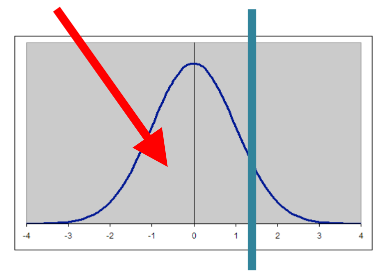
[1] From Appendix A, Field, A. (2009). *Introducing statistics using SPSS* (3rd ed.). London: Sage Publications. This is one version of such a table, which indicates both the area above and below a score. Other versions of the tables often only indicate the lower portion, still others indicate the upper portion. Some give both positive and negative Z scores, some only include positive values (remember that the normal distribution is symmetrical. This can make it confusing to consider what to look at. Drawing a diagram helps. It also might be helpful to try some different tables and find the one that makes the most sense to you. The Math is Fun website gives one option, but there are many others out there as well.

a. Suppose a population was normally distributed with a mean of 10 and standard deviation of 2. What proportion of the scores are below 12.5?

With proportion, we can think of what percentage of the scores are below 12.5. To do this, first we need to calculate the Z score associated with 12.5. Using the formula (**z = (X-M)/s**), we plug in values:

$$Z = \frac{12.5 - 10}{2} = \mathbf{1.25}$$



Then we look this up in the table. It's a positive value, and we want the scores below this. The diagram shows the area we are interested in, which corresponds with the "larger portion" column. A Z of 1.25 is associated with **.8944**. For proportion, multiple this number by 100, and round to 2 decimal points. **So 89.44% of the population is below this score.**

b. Let's say that the average IQ of a group of people is 105 with a standard deviation of 15. What is the standardized (or z- score) of someone:

   i. with an IQ of 93?
   $Z = (93-105)/15 = -12/15 = -4/5 = \mathbf{-.8}$ *(Sign matters.)*

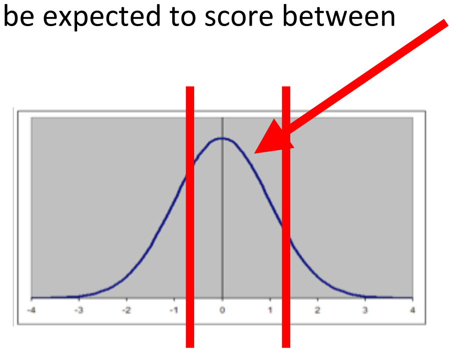
   ii. with an IQ of 135?
   $Z = (135 - 105)/15 = 30/15 = \mathbf{2}$

c. One year, many college-bound high school seniors in the U.S. took the Scholastic Aptitude Test (SAT). For the verbal portion of this test, the mean was 425 and the standard deviation was 110. Based on this information what percentage of students would be expected to score between 350 and 550?

First, calculate the two Z scores

For 350: $Z = \frac{350 - 425}{110} = -.68$

For 550: $Z = \frac{550 - 425}{110} = 1.14$



Look up the proportion of scores falling below each of these numbers (try it on the math is fun website)

For Z = -.68, proportion = .2483

For Z = 1.14, proportion = .8728

Then subtract: .8728 - .2483 = .6245

So 62.45% of the students would be expected to score between 350 and 550 on their verbal SAT.

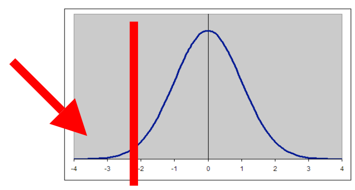
5. At Hogwarts School of Witchcraft and Wizardry, Professor Snape was concerned about grade inflation, and suggested that the school should issue standardized grades (or z-scores), in addition to the regular grades. How might this work? Harry was in four classes, each with 20 students. Harry's score, the class mean, and the class standard deviation are given below. Compute his standardized grade in each class. If we judged by standardized grades, where did he do best? Where did he do worst?

	Harry's Score	Mean	Std Dev
Care of Magical Creatures	3.80	3.75	.15
Defense Against the Dark Arts	3.60	3.25	.60
Transfiguration	3.10	3.20	.38
Potions	2.50	2.90	.75

Care of magical creatures:	(3.80 – 3.75)/.15	=	**0.33**
Defense against the dark arts:	(3.60 – 3.25)/.60	=	**0.58**
Transfiguration:	(3.10 – 3.20)/.38	=	**-0.26**
Potions:	(2.50 – 2.90)/.75	=	**-0.53**

In terms of standardized scores, Harry did best in Defense Against the Dark Arts, where he was above average in the class. Even though her best grade was in Care of Magical Creatures, the class as a whole had a higher mean, such that Harry was fairly close to the average.

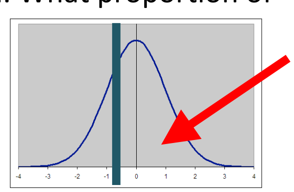
6. When the original Star Wars movie came out (1977), there was much excitement about the movie. Here are some classic problems that were considered soon after.
   a. On the average, it takes Han Solo 45 seconds to check the coordinates and make the jump into hyperspace. The standard deviation on this important task is 5 seconds. When Han and Chewbacca and their passengers are leaving for Alderaan they make the jump in 33 seconds or less. What is the probability of such an accomplishment?
   Again, first we calculate the Z score. The mean = 45, X = 33, and the standard deviation is 5. So: $Z = \frac{33 - 45}{5}$ = **-2.4**



   Look this value up in the table. Looking at the diagram, we want to know what proportion of scores fall below this – the smaller portion in the diagram. We see this corresponds with **.00820**, so there is a **.0082 probability, or a .82% chance, of this occurring.**

   b. In a space bar, there were 14 storm troopers, 3 Wookies, 9 humans, and 2 scriptwriters. An Android entered, fired a shot, and hit someone in the cheek. What is the probability that a scriptwriter was hit?
   This is just a probability question, where we have the number of scriptwriter out of the total. So 2 / 28 = .0714, or 7.14% probability that it was a scriptwriter hit.

   c. Jawas, those jewel-eyes, hooded collectors of robots and scrap, live in the desert and travel by sandcrawler. Their height is normally distributed with a mean of four feet and a standard deviation of 3 inches. The escape exit on the sandcrawler is 46 inches high. What proportion of the Jawas must duck when they use the escape exit?



   First, we need to convert things to the same scale. The mean is 4 feet, which equals 48 inches. Calculate the Z score: $Z = \frac{46 - 48}{3}$ = **-.67**
   Again, looking at the diagram, everyone over this size must duck. So we want the upper portion in the diagram. This corresponds with .74857. So **74.86% have to duck when they use the escape exit.**